Dynamic Coefficients in Hydrodynamic Bearing Analysis

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Hydrodynamic Bearing Basics

Hydrodynamic journal bearings operate by forming a thin film of oil that completely separates the shaft from the bearing surface at operating speeds. The clearance between the shaft and the bearing surface permits the formation of the thin film which the shaft rides on and as such allows the shaft center to move to one side, as shown in **Figure 1**.

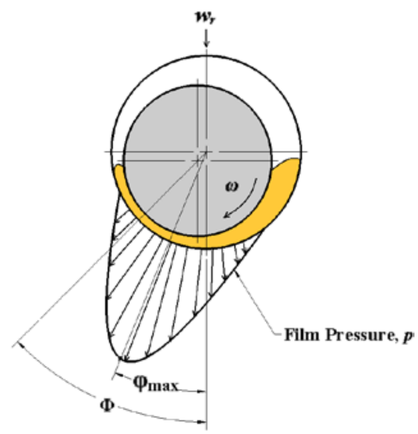

Figure 1. Pressure profile of a hydrodynamic bearing during normal operation.

The oil film thickness is a function mainly of radial load, shaft surface speed, and oil viscosity. The thickness of the oil film, among other characteristics, determines how the bearing will respond to rotating unbalance in the system at different speeds. This is important because the shaft center shown in **Figure 1** is the theoretical equilibrium point at which the formed film pressure distribution equates to the applied radial load on the bearing. In reality, rotating assemblies are not perfectly balanced and thus residual imbalance exists which causes the shaft center to orbit around the equilibrium point. The orbit frequency, amplitude, and shape are a function of the bearing geometry, operating conditions and fluid-film characteristics, all of which determine the dynamic behavior of the rotor/bearing system.

Dynamic Coefficients

To predict bearing rotordynamic behavior, fans (as an example) are modelled using the Jeffcott rotor model, shown in **Figure 2**, which assumes an unbalanced, center hung mass on an elastic shaft and two bearings modelled as a spring-mass-damper system. Likewise, each hydrodynamic bearing can be modelled as a two-dimensional spring-

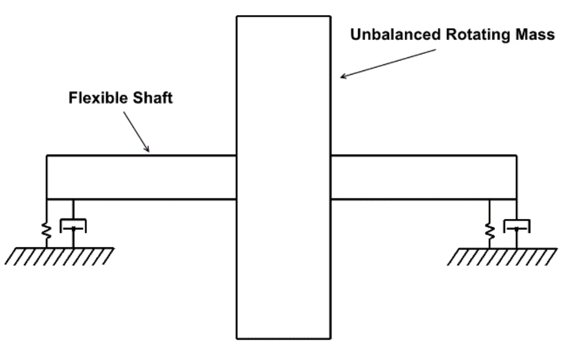

Figure 2. Jeffcott Rotor [1].

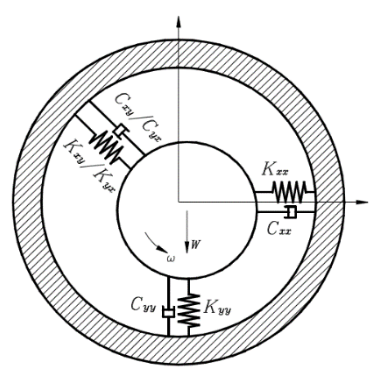

Figure 3. Spring-mass-damper model of a hydrodynamic bearing [2].

mass-damper system as shown in **Figure 3**. When analyzing bearing applications, hydrodynamic bearing software will generally produce 8 dynamic coefficients, as shown in **Figure 4**.

The *K* values represent the spring stiffness of the fluid film, which generates a force as a result of the shaft center position movement away from the equilibrium point. The *C* values represent the damping in the fluid film, which generates a force as a function of the shaft center's velocity movement away from the equilibrium point. The subscripts of the stiffness or damping (i.e. xx, xy, yx, yy) represent displacement or velocity in a direction producing a stiffness or damping force in the same or orthogonal direction.

For both stiffness and damping, there are *direct* and *cross-coupled* coefficients (and respective forces). Direct coefficients have the same directional subscripts (i.e. K_{XX}, C_{yy}, etc.) and the resultant force produced is in the same direction to the motion. For example, if the shaft center moves purely in the *y* direction, the resultant force generated by K_{yy} will be in the y direction. The effects of the direct forces are shown in **Figure 5.** Cross-coupled coefficients have different directional subscripts (i.e. C_{yx}, K_{xy}, etc.). The first subscript denotes displacement/velocity, the second subscript denotes the force direction, and the resultant force is orthogonal to the direction of motion. For example, if the shaft center moves purely in the *x* direction, the resultant force generated by K_{xy} would be in the y direction. Direct stiffness, such as K_{xx} or K_{yy}, produces a force opposite to the displacement, which can be viewed as pushing the shaft toward the equilibrium point, reducing the orbit size (and amplitude of vibration). Direct damping, such as C_{xx} or C_{yy}, produces behavior similar to K_{xx} or K_{yy}. On the other hand, cross-coupled stiffness, such as K_{xy} or K_{yx}, produces a force that is orthogonal to shaft center displacement, thus inducing a force that will tend to move the shaft 90 degrees from its direction of movement. Similarly, cross-coupled damping forces will behave as that of cross-coupled stiffness. Cross-coupled coefficients induce destabilizing effects depending on their strength and sign (+/-) that may physically translate into inducing higher vibration in the rotating assembly. Cross-coupled stiffnesses with the same sign are stabilizing, while cross-coupled stiffnesses with opposite signs are destabilizing [1]. It is important to note that for a given rotor/bearing system, destabilizing forces increase with operating speed and decease with increased bearing applied load.

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Shaft speed                                          500.0 RPM     384.8 FPM
Radial load                                          500.0 Lbs.     37.1 PSI
Thrust load                                            0.0 Lbs.      0.0 PSI
Ambient temperature                                   90.0 Deg F
Average horizontal clearance                       0.01350 Inches STD
Average vertical clearance                         0.00600 Inches STD

                        Elliptical Bore with Standard Shaft

        >>>>>>>>> Application satisfactory with ISO VG  68 <<<<<<<<<
                     Calculations based on base loading

Heat generation                 Total              0.075 HP       191.5 BTU/HR
Radial                                             0.075 HP
Maximum operating temperature                      113.0 Deg F
Radial bearing film thickness                       1.81 Mils
Attitude angle                                      47.7 Deg

Dynamic coefficients (X=horizontal direction, Y=vertical direction)
KXX =          135578.8 LB/IN              CXX =           3656.3 LB-S/IN
KXY =           86413.3 LB/IN              CXY =          -1334.5 LB-S/IN
KYX =         -378808.8 LB/IN              CYX =          -1334.5 LB-S/IN
KYY =          705200.8 LB/IN              CYY =          20042.7 LB-S/IN
    RIGID ROTOR INSTABILITY THRESHOLD IS UNBOUNDED
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Figure 4. Example Bearing Analysis output showing Dynamic Coefficients.

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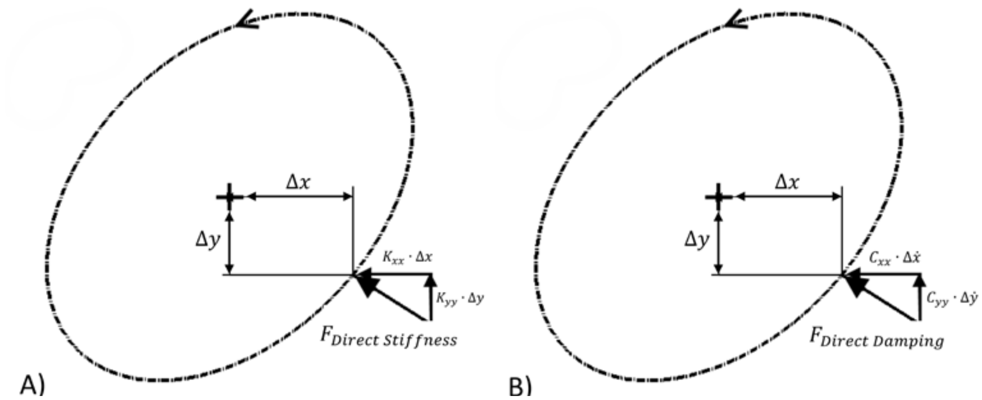

Figure 5. Forces caused by A) Direct Stiffness and B) Direct Damping and their relation to shaft orbit [2].

A rotating system is considered "stable" when stabilizing forces, such as direct stiffness and damping, exceed the destabilizing forces, i.e. the cross-coupled forces [3]. One step in predicting system stability is to determine the system natural frequencies and critical speeds. Critical speeds are the speeds where the response to an unbalance is the greatest [1]. Rotating systems can have several critical speeds, but in low- to medium-speed applications, such as fans, the operating speed is usually well below the first and subsequent critical speeds. The operating speed of a fan should not be too close to a critical speed, or the fan vibration will be unacceptably high. The most common recommendation is that the operating speed should not be within 20% above or below a critical speed.

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Effects of Bore Profile on Dynamic Coefficients

If a fan is experiencing high vibration as a result of approaching a critical speed, there are three possible options to remedy the problem: (1) change the operating speed, (2) change the load, or (3) change the dynamic properties of the bearings. Often, application requirements make options (1) and (2) unfeasible. Option (3), however, can be implemented at the bearing level on hydrodynamic bearings.

The easiest way to alter dynamic properties is to adjust the clearance between the shaft and bearing. In general, increasing the clearance lowers the operating temperature of the bearing, but also lowers the stiffness and damping coefficients.

Another way to alter the dynamic properties of the system is to change the bearing bore profile. The advantage of adjusting the bearing profile, rather than simply change the clearance, is that it allows the bearing performance to be more closely tuned to the application needs.

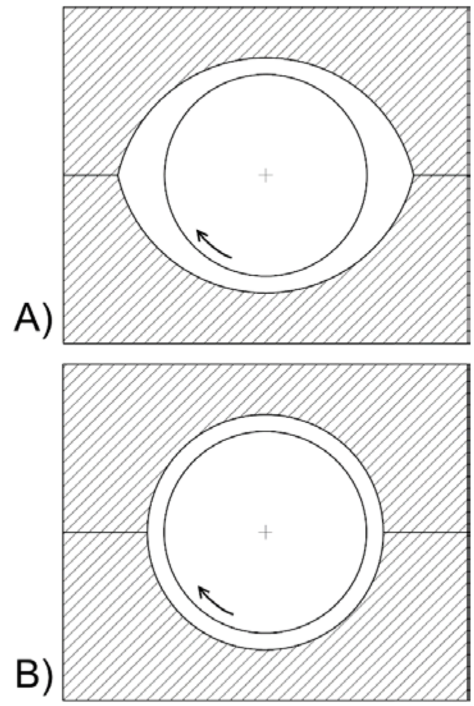

Figure 6. Common journal bearing profiles for fan applications are A) elliptical and B) cylindrical.

The two most common bore profiles of hydrodynamic bearings in fans are elliptical and cylindrical, as shown in **Figure 6**. Notice that the two figures have the same vertical clearance, but different horizontal clearance values. Using the elliptical profile allows the bearing to retain similar, if not higher, stiffness and damping to the cylindrical profile in the vertical direction, but the lower stiffness and damping in the horizontal direction allow for more shaft movement, as shown in the example bearing selection shown in **Figure 7.** However, the lower stiffness results in lower first critical speeds. Depending on the fan operating speed, this could cause higher levels of vibration than desired. If this is the case, using a cylindrical bore instead may raise the first critical speed enough to substantially lower the vibration in the fan at operating speed.

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A)

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Selection 1 of 1    Comment:
3.9375 inch Sleevoil RTL Circulating Oil Expansion Bearing
Shaft speed                                   1700.0 RPM    1753.8 FPM
Radial load                                   3000.0 Lbs.    132.5 PSI
Thrust load                                      0.0 Lbs.      0.0 PSI
Ambient temperature                             90.0 Deg F
Oil inlet temperature                          120.0 Deg F    2.50 GPM
Average horizontal clearance                 0.01300 Inches STD
Average vertical clearance                   0.00550 Inches STD

                    Elliptical Bore with Standard Shaft

      >>>>>>>>> Application satisfactory with ISO VG  68 <<<<<<<<<
                 Calculations based on base loading

Heat generation                 Total          1.365 HP      3475.7 BTU/HR
Radial                                         1.365 HP
Heat dissipation by circulating oil            1.194 HP      3039.6 BTU/HR
Oil temperature rise                             5.8 Deg F
Maximum operating temperature                  130.0 Deg F
Radial bearing film thickness                   1.68 Mils
Attitude angle                                  46.9 Deg

Dynamic coefficients (X=horizontal direction, Y=vertical direction)
KXX =         603731.3 LB/IN            CXX =          4524.9 LB-S/IN
KXY =         543395.9 LB/IN            CXY =          -530.6 LB-S/IN
KYX =       -1709715.0 LB/IN            CYX =          -530.6 LB-S/IN
KYY =        3402949.0 LB/IN            CYY =         24293.5 LB-S/IN
   RIGID ROTOR INSTABILITY THRESHOLD IS UNBOUNDED
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B)

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Selection 1 of 1    Comment:
3.9375 inch Sleevoil RTL Circulating Oil Expansion Bearing
Shaft speed                                   1700.0 RPM    1753.8 FPM
Radial load                                   3000.0 Lbs.    132.5 PSI
Thrust load                                      0.0 Lbs.      0.0 PSI
Ambient temperature                             90.0 Deg F
Oil inlet temperature                          120.0 Deg F    2.50 GPM
Average horizontal clearance                 0.00650 Inches NON-STD
Average vertical clearance                   0.00650 Inches NON-STD

                    Cylindrical Bore with Standard Shaft

      >>>>>>>>> Application satisfactory with ISO VG  68 <<<<<<<<<
                 Calculations based on base loading

Heat generation                 Total          1.606 HP      4087.8 BTU/HR
Radial                                         1.606 HP
Heat dissipation by circulating oil            1.433 HP      3647.6 BTU/HR
Oil temperature rise                             7.0 Deg F
Maximum operating temperature                  132.0 Deg F
Radial bearing film thickness                   1.67 Mils
Attitude angle                                  56.5 Deg

Dynamic coefficients (X=horizontal direction, Y=vertical direction)
KXX =        1459274.0 LB/IN            CXX =         10395.8 LB-S/IN
KXY =         829864.7 LB/IN            CXY =         -9238.0 LB-S/IN
KYX =       -3508671.0 LB/IN            CYX =         -9238.0 LB-S/IN
KYY =        2691131.0 LB/IN            CYY =         37828.3 LB-S/IN
   RIGID ROTOR INSTABILITY THRESHOLD =     6505.51 RPM
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Figure 7. Example bearing selections comparing a) elliptical bore and b) cylindrical bore.

Conclusion

Unlike rolling element bearings, hydrodynamic bearings produce both stiffness and damping forces, which provide significant flexibility for machine designers. When selecting hydrodynamic bearings for an application, it is critical to consider the dynamic effects to ensure that the system operates as desired. In some applications, the bearing load, shaft speed, and oil viscosity can be easily changed to fine tune the dynamic performance. However, in tightly constrained or existing applications, it may be preferable to alter the bearing bore profile to find the best balance between the dynamic properties and the critical speeds of the system.

References:

[1] Vance, J., Zeidan, F., Murphy, B., 2010, *Machinery Vibration and Rotordynamics,* John Wiley & Sons, Inc. Hoboken, NJ, Chap. 3.

[2] He, M., Cloud, C.H., and Byrne, J., 2005, “Fundamentals of Fluid Film Journal Bearing Operation,” *Proceedings of the Thirty-Fourth Turbomachinery Symposium,* Texas A&M University, pp. 155-176.

[3] Moore, J.J., n.d., “Rotordynamics Tutorial: Theory, Practical Applications, and Case Studies,” Southwest Research Institute.

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